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Experiments involving mirror transponders in rotating frames

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Abstract. Two experiments are described in which mirrors were mounted on rotating turntables and measurements were made of the frequency of the reflected radiation. The order of accuracy was one part in 10^{17}, which exceeded that required to detect second-order relativistic phenomena. It is shown that the mirrors are equivalent to transponder systems and that the application of logical arguments to the telemetered observations confirms earlier theoretical predictions of the radius and angular velocity of a rotating system measured entirely from a point within that system.

1. Introduction

Jennison (1964) showed by deductions from the results of an experiment by Champeney and Moon (1961) that the radius of a rotating system, measured from a single domain rotating with the system, would be $r_r = r(1-\omega^2r^2/c^2)^{1/2}$ and that the angular velocity would be $\omega_r = \omega(1-\omega^2r^2/c^2)^{-1/2}$, where ω and r are the angular velocity and radius in the non-rotating laboratory frame and where c is the local velocity of light. Despite the logic of the original argument, it was considered advisable to seek confirmation in a suitably designed laboratory experiment. It transpires that all the information required for verifying the transformations is available from this and other experiments, but it can be shown (Ashworth and Jennison 1975, submitted for publication) that the results may be derived by the application of the principles of special relativity. In this paper it will be shown that if the radius at the rotating observer did not contract relative to that at the centre then the results of either the Mössbauer experiment or the following transponder experiment would differ from the observations.

The two experiments described herein, together with a third experiment already described (Jennison and Davies 1974) investigate the behaviour of mirrors in rotating systems. In analysing reflection at a mirror it is usual to assume lossless and instantaneous reflection. However, if we consider the mechanism involved it becomes apparent that the reflection process involves the absorption and re-emission of the wave by the conduction electrons.

Any electromagnetic wave incident on the surface of a conductor penetrates a certain distance inside the conductor. The depth of penetration is determined by the skin effect. The wave induces a current in the conductor which in turn re-radiates a wave. In the case of a perfect conductor the re-radiated wave has the same amplitude as the incident wave. The mirrors used in the following experiment were not perfect conductors, however. When a current is induced by a wave incident upon an imperfect conductor there are losses in the material and the re-radiated wave has a smaller amplitude than the incident wave.

In a macroscopic system the losses correspond to the requirement of a transponder ranging system: that the observer shall be able to extract a small amount of energy in order to measure the pulse repetition frequency, the carrier frequency and other parameters of the radiation that he may wish to assess. Mirrors are transponder systems receiving the incoming signals and re-radiating them without change of frequency in the local frame. This conservation of frequency was demonstrated experimentally to a very high order of accuracy in our verification of the absence of a second-order Doppler shift for light incident normally on a transversely moving mirror (Jennison and Davies 1974). It is necessary for the present purposes, however, to demonstrate that the same property holds when the mirror is constrained to move in a circular arc normal to the incident radiation.

2. Experiment 1

The apparatus is shown in figure 1. It is similar to that used in the earlier experiment to show that there is no frequency shift from a transversely moving mirror (Jennison and Davies 1974). In this case the laser beam is directed onto a 45° mirror at the centre

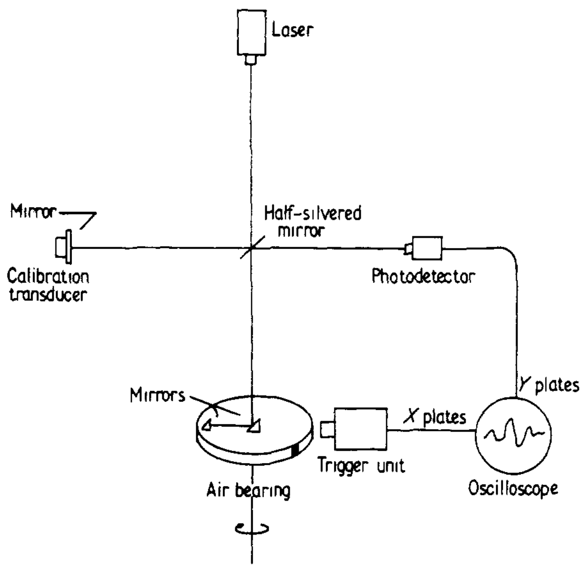

Figure 1. The arrangement of the apparatus for experiment 1.

of the rotating table and from this mirror it is reflected onto a vertical mirror at the periphery. The returned signal is reflected from a half-silvered mirror to a photodetector where it combines with a reference beam to produce interference fringes. As in the previous experiment the display on the oscilloscope is synchronized to the rotation of the table, and the reference beam can be calibrated by applying a small first-order Doppler shift from a piezoelectric transducer. Movement of the calibration transducer produces movement of the interference pattern which is displayed on the oscilloscope and confirms that the equipment is functioning with sufficient accuracy.

The rotating table consists of an air bearing which is spun up and then allowed to freewheel, with the drive removed. As the table gradually slows down the display on the oscilloscope is observed and changes in the fringe pattern are recorded. Typical observation periods were of the order of six minutes, and typical fringe shifts observed were ± 4 whereas second-order relativistic fringe shifts would have been of the order of $+60$. The radius in the laboratory system was 10·5 cm and the maximum angular velocity of the disc was 100 rad s^{-1}.

This experiment shows that any frequency shift in the beam reflected from the moving mirrors is less than two parts in 10^{17}, which is better than one order of magnitude less than any predicted relativistic frequency shift.

3. Experiment 2

A diagram of the experiment is shown in figure 2. It was designed to show that, for a signal sent from the laboratory frame to rotating observers and reflected such that incident and reflected beams are tangential to the disc, no relativistic frequency shift is produced.

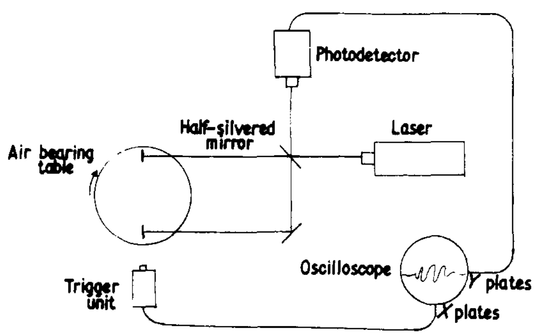

Figure 2. The arrangement of the apparatus for experiment 2.

The reference beam of the interferometer is sent directly to the detector while the other beam is reflected in turn from a mirror moving towards the beam and then from a mirror moving away from the beam. Interference with such a large path difference is possible because of the great coherence length of the laser. The double reflection in the second beam is designed to minimize first-order Doppler effects, but because any time dilatation effects which may be produced would be independent of the direction of the velocity of the mirror, relativistic effects would be additive rather than subtractive.

Synchronization and display systems operate as already described (Jennison and Davies 1974). As in the two previous experiments the rotating table is initially spun up and then allowed to freewheel. Here typical observation periods were of the order of five minutes, and typical observed fringe shifts varied from $+3$ to $+5$.

Fringe shifts, predicted on the assumption that relativistic shifts were present, were of the order of $+150$.

This experiment shows there is no relativistic frequency shift to an accuracy of better than 5%.

4. Implications of the experiments

We now consider the implications of these experiments. First the argument for the contraction of the radius as measured by a rotating observer, mentioned in the introduction, is produced in more detail to show the role of experiment 1.

In accordance with modern methods of laboratory metrication (Sanders 1965), which employ either radar or long-range reflection interferometry, the radius CB measured from the centre C of a rotating system is

$$r = \tfrac{1}{2}c(t_r - t_t) = \tfrac{1}{2}cN_C$$

where N_C is the number of local units of time (ticks of a proper clock) in the interval $(t_r - t_t)$ between transmission and reception at the centre of either a pulse or an interfering wave returned from the periphery. If the pulse, or wave, on reception at C, is immediately transmitted back to the point B, a similar measurement may be performed in the following interval, N_C. Thus a pulse repetition frequency of $1/N_C$ will maintain a continuous measurement of the radius from the centre. A similar operation may be performed by an observer at B. It would not be necessary for him to install additional equipment for he may measure the radius by timing the pulse repetition frequency of his responses to the same sequence of pulses on his own local clock. Let this pulse repetition frequency be $1/N_B$. The measurement of radius performed by B could be determined unambiguously by C if the value of N_B could be telemetered back to C. But N_B is the number of local units of time at B in the intervals between transmission and reception at B. If both the intervals between clock ticks and the intervals between radar pulses are telemetered back to C, both should be equally transformed in the process and the ratio will be conserved. It was pointed out by Jennison (1964) that the information rate is subject to a second-order Doppler shift. Thus both the radar period and the tick period will be received at C shifted to the red by the same factor, $(1-\omega^2r^2/c^2)^{1/2}$. The telemetered but uncalibrated pulse repetition frequency at B is received at C as C's own pulse repetition frequency, N_C. Calibrating this in the ticks of the telemetered clock, we find $N_B = N_C(1-\omega^2r^2/c^2)^{1/2}$. It does not, therefore, seem to be possible to avoid the conclusion that B will measure a radius:

$$r_B = \tfrac{1}{2}cN_C(1-\omega^2r^2/c^2)^{1/2} = r(1-\omega^2r^2/c^2)^{1/2} \tag{1}$$

We now consider the substantive assumptions implicit in the derivation of equation (1).

(i) That the velocity of light is locally invariant

(ii) That signals from a standard clock, or a sequence of events timed by such a clock, at the centre will be received at the periphery at a faster rate, $(1-\omega^2r^2/c^2)^{-1/2}$ when compared to the local standard (or 'proper') clock.

(iii) That signals from a standard clock, or a sequence of events timed by such a clock, at the periphery will be received at the centre at a slower rate, $(1-\omega^2r^2/c^2)^{1/2}$, when compared to the local standard.

(iv) That in their own local frames transponders at the centre and at the periphery transmit signals at the same frequency as that at which they are received and hence that a signal from the centre will be returned by the peripheral transponder without change of frequency measured in the central frame.

(v) That the distance defined by equation (1) is a valid measurement of this parameter.

(vi) That no events are lost at the transponders, or in transit.

We justify assumptions (i) to (v) by the following experimental evidence.

(i) The Michelson–Morley and subsequent experiments.

(ii) and (iii) The experiments of Champeney *et al* (1965) and also Farley *et al* (1968).

(iv) Experiment (1) described above.

(v) The expression corresponds to the radar distance which is currently used for precise measurement and it may also be identified with the standardization and precise metrication of length. The standard of length is defined (Sanders 1965) as 1650763·73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $^2p_{10}$ and $^5d_5$ of the atom ^{86}Kr. This is employed in reflection interferometry for the metrication of length and equation (1) is tacitly assumed.

Assumption (vi) has been mentioned by Essen (1971) in other contexts. However we disagree with Essen, for unless a physical mechanism is proposed for the loss or gain of events, the interpretation of physical experiments must rely on the laws of physics as they are known at the present time. On these grounds our assumption is considered justified.

In order to illustrate more clearly why the radius r_r contracts, consider the '*Gedanken*' experiment often used by texts on special relativity (see, for example, Rosser 1967).

In figure 3, A sends a signal to B which is reflected back to A. Σ shows the situation as seen by B. Σ′ shows the situation as seen by A. By the usual method, if we assume $y_0 = y_0'$ we can prove $t_0' = t_0(1-v^2/c^2)^{1/2}$, and if we assume this time dilatation we can prove $y_0 = y_0'$.

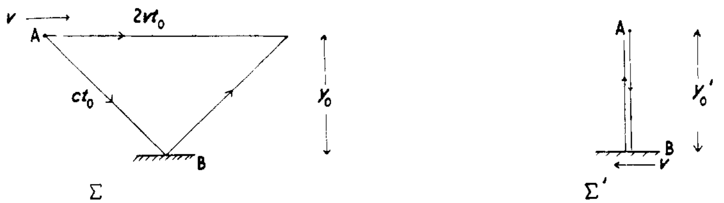

Figure 3. Light emitted by A and reflected by B as seen in Σ and Σ′ where Σ is the rest frame for B and Σ′ is the rest frame for A.

y_0 = distance between A and B measured in Σ
y_0' = distance between A and B measured in Σ′
v = relative velocity
t_0 = $\frac{1}{2}$ time from transmission to reception of a signal measured in Σ
The interval t_0' is not marked on the diagram but corresponds to half the time from transmission to reception of a signal measured in Σ′

Consider now the case where one observer rotates around another as shown in figure 4. If observer A were to follow the path EC we would have the same conditions as above, but A is in fact constrained to follow path DC, and if we assume

$$t(A) = t(B)(1-v^2/c^2)^{1/2}$$

then $y_0 \neq y_0'$, in fact

$$y_0' = y_0(1-v^2/c^2)^{1/2}.$$

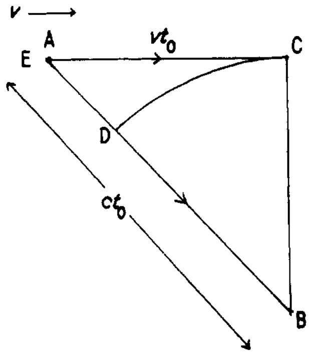

Figure 4. Comparison of rotating and linear paths. The diagram is depicted in the rest frame of the centre and corresponds, in the case of rectilinear motion, to the left-hand side of figure 3

Whence, in the present context,

$$r_B = r(1 - r^2\omega^2/c^2)^{1/2}$$

in agreement with equation (1). But if we now accept equation (1) and let A and B assign equal and opposite velocities to each other as required in special relativity we have

$$v = \omega r = \omega_B r_B. \tag{2}$$

Substituting from equation (1) we obtain

$$\omega_r = \omega\left(1 - \frac{\omega^2 r^2}{c^2}\right)^{-1/2} \tag{3}$$

This is substantiated by another argument for the transformation of angular velocity, which follows from experimental results. The Mössbauer experiments (Champeney *et al* 1965) have shown that when a standard atomic clock is caused to revolve at radius r about a distant centre of rotation, the clock may be recorded at that centre (eg by telemetry) and will be found to tick more slowly than an identical clock at the centre. This is an equivalent statement to the existence of the transverse Doppler shift, the effects are the same in this context and the dilatation factor is $(1 - v^2/c^2)^{1/2}$, where $v = \omega r$. Let the centre of rotation lie on a line in the baseplate which does not participate in the rotation; this line can be fixed in the direction of a distant star. If an observer revolves with the moving clock he can transmit a single pulse to the centre each time that he crosses the line and continue this for a very large number, N, of transits so that a long series of pulses is telemetered together with the ticking of his standard clock. Both the frequency of the ticking of the clock and the frequency of the repetition of the pulses will be transverse Doppler shifted when received at the centre so that n_r, the number of ticks between successive pulses, is conserved in the process of transmission. Upon the completion of the Nth transit the central observer will agree that there were N transits, and he can determine the rotational period in his system by dividing the total time that has elapsed on his local standard clock by N, thus deriving n_c ticks per revolution. He can determine the rotational period that was measured by the distant revolving observer by consulting his telemetry record and he obtains the answer n_r. The revolving observer therefore determines the period to be shorter than that measured by the central observer by the ratio n_r/n_c, that is by the factor $(1-v^2/c^2)^{1/2}$. Conversely, the frequency of revolution recorded by the revolving observer is greater than that recorded by the central observer by $(1-v^2/c^2)^{-1/2}$. Both observers will claim that *in their own small localities* there are 2π radians in one revolution, so that the angular velocity determined by the revolving observer, ω_r, is related to that obtained by the central observer, ω, by

$$\omega_r = \frac{\omega}{(1-v^2/c^2)^{1/2}}$$

in agreement with equation (3).

The difference between the angular velocities ω and ω_r may be shown to agree with the value for the Thomas precession rate.

If the observer is freely pivoted, experiencing no constraining torque from other objects participating in the rotation (eg neighbouring parts of a rotating disc) and was not given such a torque in a previous epoch, in particular in the initial spinning-up of the system, then he will act as a free compass and retain his orientation relative to the fixed stars. The direction of the centre of the system will then appear to him to rotate relative to the fixed stars at ω_r. If he is bonded to the rotation or if he is perfectly free but has once been given a torque to bring him into synchronous rotation with his environment then it will appear that the universe rotates relative to him at $-\omega_r$.

The observations of the freely pivoted revolving observer and the central observer may be related by an expression in the form of an incremental angular velocity (Møller 1972)

$$\boldsymbol{\omega}_T = (-1/v^2)[(1-v^2/c^2)^{-1/2}-1](\boldsymbol{v}\times\dot{\boldsymbol{v}}) \tag{4}$$

which is the Thomas precession. In equation (4), since $\boldsymbol{v}\times\dot{\boldsymbol{v}}$ is a constant vector perpendicular to $\boldsymbol{v}$ and $\dot{\boldsymbol{v}}$, one may substitute $v^2\boldsymbol{\omega}$ for $(\boldsymbol{v}\times\dot{\boldsymbol{v}})$. Equation (4) then becomes

$$\boldsymbol{\omega}_T = -\boldsymbol{\omega}[(1-v^2/c^2)^{-1/2}-1] = \boldsymbol{\omega}-\boldsymbol{\omega}(1-v^2/c^2)^{-1/2}$$

whence, from equation (3), since $v = \omega r$,

$$\boldsymbol{\omega}_T = \boldsymbol{\omega}-\boldsymbol{\omega}_r. \tag{5}$$

The spatial direction given by the distant stars when referred from one observer to the other therefore exhibits a precession simply because they each obtain a different angular velocity for the common rotating system, in this case the disc, relative to the same fixed stars.

5. Conclusions

We have described two experiments which investigate the behaviour of mirrors in rotating systems. By considering these mirrors as transponders we have shown that these experiments; together with others as listed, substantiate the note by Jennison (1964) and imply that: (i) the radius of a rotating disc, measured by an observer on the circumference of the disc is

$$r_r = r\left(1-\frac{r^2\omega^2}{c^2}\right)^{1/2}$$

where r_r, r, ω, c are as defined in the introduction, and as a necessary result of this: (ii) the angular velocity measured by the observer is

$$\omega_r = \omega\left(1 - \frac{r^2\omega^2}{c^2}\right)^{-1/2}$$

The equation for angular velocity was also derived independently from an argument involving clock rates, and it was then shown that the difference between the angular velocity of the disc measured at the centre and that measured at the circumference is equal to the Thomas precession.

These transformations show that if an observer moves out across a uniformly rotating system, his distance to the centre contracts as he passes beyond the inertial radius $r = c/(\omega\sqrt{2})$, and approaches zero when the inertial radius is the maximum c/ω, but his angular velocity continues to increase, tending to infinity at the singularity $r = c/\omega$.

For distances large compared to the radius of the disc, in the directions described, it may be shown from the experimental results that distances measured by the rotating observer are contracted by $(1 - r^2\omega^2/c^2)^{1/2}$ compared with distances measured by the observer in the laboratory frame. For very small distances ($\ll$ radius) the geometry is similar to that for rectilinear motion and the contraction is less severe. In the limit, for infinitesimal distance from the observer on the disc, there is no contraction normal to the motion.

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